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Large magnetic field instabilities induced by magnetic dipole transitions

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We present a new mechanism that will limit very high magnetic fields which have been conjectured to exist in connection with some astrophysical phenomena. Low lying strongly interacting particles and resonances mixing with each other via magnetic dipole QED couplings force a vacuum instability for large external magnetic fields. These mixings limit fields to a few GeV².

It has been realized for a long time that a constant electric field is quantum mechanically unstable [1,2] in that, due to tunneling, it will emit charged particle–antiparticle pairs. This rate does not become appreciable unless the field E is large enough, in appropriate units, $|eE| \sim m^2$; e is the charge of the particles and m is their mass. Due to the above restriction this process is primarily of academic interest. Constant magnetic fields on the other hand are stable against emission of spin zero or spin one-half pairs. It has been noted [3] that vector particles with an anomalous magnetic moment will induce an instability. Again, the magnetic fields required are enormously large; however, it has been conjectured that such large magnetic fields do accompany various astrophysical phenomena as supernovae [4] with fields of the order of 10^{10} T, cosmic strings [5] with fields $\sim 10^{16}$ T and other cosmic objects. In this article we point out that the existence of a magnetic transition moment between a spin zero and a spin one particle will likewise precipitate an instability in a sufficiently large

magnetic field. This novel mechanism is interesting per se but the interest is more than academic since it destroys fields at much lower values than the mechanism proposed in ref. [3]. The characteristic value is several orders of magnitude smaller.

We take the simplest model which will exhibit the above effect; it consists of a neutral scalar field $s(x)$ with mass μ and a vector field $v^\mu(x)$ with mass M and with parity and charge conjugation opposite to that of s . An electric dipole transition couples the electromagnetic field to s and v . The lagrangian for this model is

$$\begin{aligned} \mathcal{L} = & \frac{1}{2} \partial_\mu s \partial^\mu s - \frac{1}{2} \mu^2 s^2 \\ & - \frac{1}{4} (\partial_\mu v_\nu - \partial_\nu v_\mu) (\partial^\mu v^\nu - \partial^\nu v^\mu) + \frac{1}{2} M^2 v_\mu v^\mu \\ & - \frac{1}{2} \frac{e}{A} s (\partial_\mu v_\nu - \partial_\nu v_\mu) F^{\mu\nu}, \end{aligned} \quad (1)$$

$F^{\mu\nu}$ is the electromagnetic field strength tensor and the last term is the aforementioned magnetic dipole coupling. A has the dimensions of a mass and its magnitude provides the strength of this coupling. For $F^{\mu\nu}$ constant we can easily find the eigenmodes of this lagrangian. In momentum space the equations of motion are

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$$\mathcal{M}(p) \begin{bmatrix} s(p) \\ v_\nu(p) \end{bmatrix} = 0, \quad (2)$$

where the matrix $\mathcal{M}(p)$ is

$$\mathcal{M}(p) = \begin{bmatrix} p^2 - \mu^2 & -i(e/A)p_\lambda F^{\lambda\nu} \\ i(e/A)p_\lambda F^{\lambda\mu} & -g^{\mu\nu}(p^2 + M^2) + p^\mu p^\nu \end{bmatrix}. \quad (3)$$

The condition $\partial \cdot v = 0$ implies that the eigenvectors must be orthogonal to $[0, p_\nu]$. There are two modes of the form $[0, \epsilon_\nu(p)]$, with $\epsilon(p) \cdot p = 0$ and $p_\lambda F^{\lambda\nu} \epsilon_\nu(p) = 0$; these describe two, mass M polarization directions of the vector meson. The scalar and vector particle couple in eigenstates of the form $[s(p), iv(p)p^\lambda F_{\lambda\nu}]$. The solution of

$$(p^2 - \mu^2)(p^2 - M^2) + \frac{e^2}{A^2} p_\lambda p_\eta F^{\lambda\nu} F^{\eta}_{\nu} = 0 \quad (4)$$

yields the dispersion relation for these modes.

We now specialize to a constant magnetic field pointing in the z direction

$$F^{12} = -F^{21} = H,$$

and all other components vanish. Eq. (4) takes the form

$$(p^2 - \mu^2)(p^2 - M^2) - \frac{e^2}{A^2} \mathbf{p}_\perp^2 H^2 = 0; \quad (5)$$

$\mathbf{p}_\perp^2 = p_x^2 + p_y^2$. The energies satisfy

$$p_0^2 = \mathbf{p}^2 \pm \left(\left[\frac{1}{2}(M^2 - \mu^2) \right]^2 + \frac{e^2}{A^2} \mathbf{p}_\perp^2 H^2 \right)^{1/2}. \quad (6)$$

For sufficiently large magnetic fields the lower solution becomes negative indicating an instability. For $p_z = 0$ this occurs whenever

$$\frac{e^2}{A^2} H^2 \geq \mathbf{p}_\perp^2 + M^2 + \mu^2 + M^2 \mu^2 / \mathbf{p}_\perp^2.$$

There will be a range in \mathbf{p}_\perp for which the above is satisfied whenever

$$H \geq H_c = \frac{(M + \mu)A}{|e|}. \quad (7)$$

The instability will manifest itself in that fields $H \geq H_c$ will create pairs of the s and v particles. Γ , the decay rate per unit time per unit volume of a “vac-

uum” with such a large field may be calculated by standard methods [2]

$$\Gamma = \text{Re} \int \frac{d^4 p}{(2\pi)^4} \ln \det \mathcal{M}(p), \quad (8)$$

where $\mathcal{M}(p)$ is defined in eq. (3). It is only for $H \geq H_c$ that the integral in eq. (8) develops a real part; the result is

$$\Gamma = \frac{1}{96\pi} \frac{eH}{A} \left[\left(\frac{eH}{A} \right)^2 - 2(M^2 + \mu^2) + \left(\frac{(M^2 - \mu^2)A}{eH} \right)^2 \right]^{3/2} \quad (9)$$

The question remains as to whether and when this mechanism will become operative. As mentioned earlier, in pure QED magnetic fields are stable. Thus, magnetic instabilities must be due to other interactions. In ref. [3] it is the anomalous magnetic moment, a residue of a non-abelian gauge group, that is responsible for the instability. What we have in mind is QED together with QCD. As this combined theory is far too intractable we consider a low energy truncation into an effective lagrangian involving low energy mesons and resonances. Among the low lying hadrons [6] there are several scalar and vector meson combinations, satisfying the parity and charge conjugation requirements for our analysis to be valid. For example, we could consider the π meson and the $h_1(1170)$ and as the h_1 is seen to decay to a π and a ρ vector dominance insures an $\pi h_1 \gamma$ coupling of the type as envisioned in eq. (1). The masses of these particles are at or below a GeV. We expect A , the mass controlling the strength of the magnetic dipole transition, likewise to be of the order or below a GeV. The critical field calculated in eq. (7) will be of order a few GeV^2 and therefore significantly smaller than the fields discussed in ref. [3]. Fields above 10^{15} T will be destroyed by particle emission. As in the real theory these fields correspond to particles that are bound states of quarks the question may arise as to whether such bound states will survive such large magnetic fields, or put in another way, whether the effective theory we have discussed will remain valid in the presence of such large fields. The strengths of magnetic fields at which we expect an effective theory to break down are such that the Zeeman energies are of the order of hadronic level separation or again

for fields such that $eH \sim \text{GeV}^2$; the same order as the critical fields necessary for the instability we have discussed. As these fields are of the same order of magnitude we cannot conclude whether this instability will or will not occur. The critical fields needed to induce this instability are *not* significantly larger than the fields that would make the effective theory invalid. (Naively, we could try to apply our analysis to 0^- and 1^+ pairs of positronium states and claim an instability for $eH > 2m_e^2$; the Zeeman energies in such fields are much larger than the energy differences in positronium which are of the order of $\alpha^2 m_e$.) We hope to return to these points in a future work. Our considerations also apply to radial magnetic fields, which are of interest in the case of cosmic strings.

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